

Ph666: Quantum Field Theory
Problem Set 3: Gauge theory and Symmetry Breaking

(1) Review and summarize Peskin's derivation of the beta function in SU(N) gauge theory. If Λ_{QCD} , the scale at which the coupling α_s diverges (according to the one loop beta function), is 200 MeV, compute the value of α_s at the charm quark mass (1.5 GeV). How would you compute α_s at the b mass (4.5 GeV)?

(2) Consider a real scalar field $\vec{\phi} = (\phi^1, \phi^2, \phi^3)$ with Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \vec{\phi}) \cdot (\partial^\mu \vec{\phi}) - V(|\vec{\phi}|) \quad .$$

(a) Show that this model has an SO(3) symmetry.

(b) Suppose that the minimum of the potential V is at a non-zero value of $|\vec{\phi}|$. Describe the symmetry properties of the classical groundstate and count the number of Goldstone bosons.

(c) If the potential is given by

$$V = -\frac{1}{2}m^2 \vec{\phi} \cdot \vec{\phi} + \frac{\lambda}{4}(\vec{\phi} \cdot \vec{\phi})^2$$

with $m^2, \lambda > 0$, work out the classical particle spectrum by perturbing about the minimum.

(3) Suppose the SO(3) of the model in problem (2) is a gauge symmetry. Write out the Lagrangian explicitly. Derive the particle spectrum that results from the Higgs mechanism when the minimum of the potential V breaks the gauge symmetry. (Hint: consider rewriting the SO(3) symmetry as an SU(2) symmetry.)