

**Ph665: Quantum Field Theory**  
**Problem Set 3: Functional Methods**

(1) Calculate the transition amplitude  $\langle x_2, t_2 | x_1, t_1 \rangle$  for a non-relativistic free particle using the path integral.

(2) Consider a free scalar field in  $d$  Euclidean dimensions. Let the action be

$$S_0[\phi] = \int d^d x \phi K \phi$$

where  $K = -\partial^2 + m^2$ .

(a) Calculate the partition function  $Z$  and the correlator  $\langle \phi(x)\phi(0) \rangle$ .

Now consider the case  $m = 0$ .

(b) Give a scaling argument that

$$\langle \phi(x)\phi(0) \rangle \propto |x|^{2-d}$$

(c) Show that for  $d = 3$

$$\langle \phi(x)\phi(0) \rangle = \frac{1}{4\pi|x|}$$

(d) Show that for  $d = 2$

$$\langle \phi(x)\phi(0) \rangle = \frac{1}{2\pi} \ln|x|$$

(3) Do problem 9.1 (a) and (b) in Peskin.

(4) Bonus Problem: Consider QED in the background of a constant potential field  $A_0$ . Compute the correction to the vacuum energy at leading order as a function of  $A_0$ .