Stephen D.H. Hsu

DOE Research Report 2003-2004

(1) QCD under extreme conditions

(2) Holography and Extra dimensions

(3) Dark Energy

1 QCD under extreme conditions

Positivity and Fermionic Dense Matter, with D.K. Hong, to appear in the proceedings of 21st International Symposium on Lattice Field Theory (LATTICE 2003), Tsukuba, Ibaraki, Japan, 15-19 Jul 2003, hep-lat/0309103.

Positivity and Dense Matter, with D.K. Hong, Phys.Rev.D68: 034011, 2003, hep-ph/0304156.

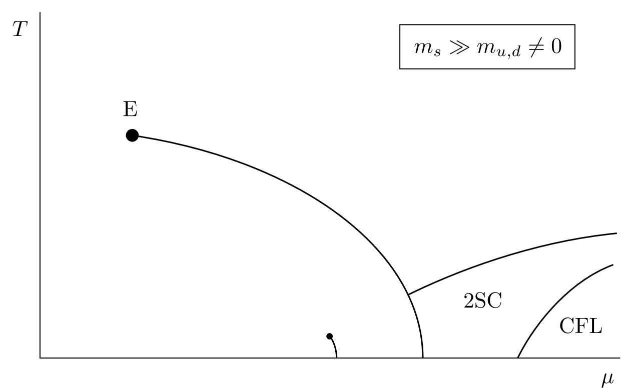

Figure 1: Phase diagram of QCD in the temperature-density plane.

(A) Color Superconductivity: a new state of matter. Our group showed that the true vacuum of matter at high density is a color superconductor, first through a dynamical calculation (1999), and now using rigorous theorems: anomaly matching plus positivity result.

There are interesting implications for neutron stars, phase diagram of QCD. Useful laboratory for theoretical study.

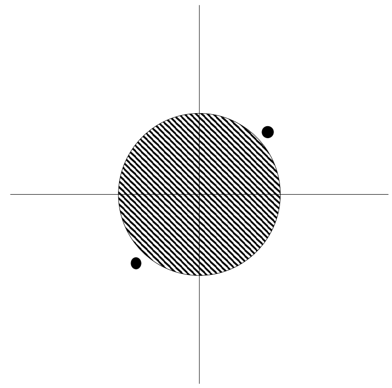

Figure 2: Excitations near a Fermi surface.

(B) Dense matter effective theory: effective field theory of modes near a Fermi surface.

We showed that the Euclidean quasiparticle determinant of the effective theory is positive and real. This means that the dynamics at asymptotic density can be simulated using lattice methods. The result also allows us to prove rigorous QCD inequalities governing the low-energy spectrum of QCD at high density.

D.K. Hong and S. Hsu, Phys.Rev.D66:071501,2002 (hep-ph/0202236), Phys.Rev.D68:034011, 2003 (hep-ph/0304156) and LATTICE 2003 (hep-lat/0309103).

Sign Problem in QCD at finite density. Precludes lattice simulation and rigorous theorems. (Also an issue in condensed matter: many body simulations.)

How fundamental? fermions = sign problem? But, zero-density QCD has no sign problem.

Euclidean Path Integrals and Importance Sampling:

Partition function

$$Z = \int \prod_{\{x\}} dA_x \, d\bar{\psi}_x \, d\psi_x \;\; e^{-\beta S[A,\bar{\psi},\psi]}$$

Multi-dimensional integral ($> 10^8$!) cannot be evaluated directly.

Use techniques like Monte Carlo with importance sampling.

For importance sampling, want integrand which can be interpreted as probability distribution (real, positive).

Worst case:

$$Z = \sum (\; + \;\; - \;\; + \;\; - \;\; + \;\; - \;\; + \;\; \cdots)$$

Luckily, lattice QCD integrand positive and real at zero baryon density. But not true at finite chemical potential.

Finite density $\longleftrightarrow$ Fermi surface. But we showed there is No Sign Problem for excitations near the Fermi surface. Use HDET = High Density Effective Theory.

Intuition: curvature of FS not evident to low energy modes. Fermi sea looks like Dirac sea.

This observation leads to possible methods for lattice simulation, and some rigorous theorems for QCD at asymptotic density. Can show that CFL phase is the unique groundstate.

(C) Application to cold atoms: Hong and I applied the dense effective field theory to cold atom systems. We have confirmed the recent prediction by condensed matter theorists that the critical temperature for superfluidity induced by a Feshbach resonance can be of order the Fermi temperature.

High Temperature Superfluid and Feshbach Resonance, cond-mat/0302422, submitted to PLA.

2 Holography and Extra Dimensions

Holography, Entropy and Extra Dimensions, with D.K. Hong, hep-ph/0308290.

Higher dimensional models (brane worlds) are in conflict with holographic and black hole information (entropy) bounds.

Entropy bounds limit the amount of information that can be contained in a region V. Surprising, a limit can be derived which depends on the area A of the boundary $B = \partial V$ in Planck units. This constrains the fundamental scale of quantum gravity, forcing it to be much larger than $\sim$ TeV - a hierarchy relative to the weak scale is inevitable.

Holography: A hint from black holes

• Bekenstein, Hawking and black hole entropy. Using the Hawking temperature and $dS = dQ/T$, one obtains

$$S = \frac{A}{4}$$

in Planck units. Note: direct calculations of entropy establish that units of A are determined by UV cutoff scale.

• Generalized Second Law (GSL): combined entropy

$$S = S_{BH} + S_{\text{Exterior}}$$

cannot decrease.

• Susskind's gedanken experiment: consider region V of size R and energy less than that of a black hole of radius R:

$$E(V) \ < \ M_{BH}(R) \quad .$$

Now imagine a spherical shell of mass $M_{BH}(R) - E(V)$ which collapses in on the region V. This produces a black hole of mass $M_{BH}(R)$ and entropy $S = A/4$, where $A = \partial V$. By the GSL, this entropy is larger than the entropy of the original region:

$$S(V) \ < \ \frac{A}{4} \quad .$$

Flat space thermodynamics fails: why?

• This is highly counterintuitive. Usually think of entropy S as an extensive quantity which grows like the volume V: $\boxed{S \sim V}$

• Thermal system: entropy density

$$s = S/V \sim T^3 \quad .$$

• Since

$$S \sim \text{Log} \; (\# \; accessible \;\; quantum \;\; states \;) \; ,$$

we have

$$\# \; accessible \;\; quantum \;\; states \;\; \sim \;\; e^S \; < \; e^{A/4} \;\; \neq e^V \; !$$

• Information (e.g., in bits) required to specify state of a system bounded by $A/4$, rather than V.

The maximum amount of information contained in a spacelike region is determined by the area A rather than the volume.

• When we ignore gravity, we overcount the number of states! ['t Hooft, 1993]

Holography in D dimensions: gravity and state counting

• Generalize ’t Hooft’s result. Compute entropy of a region of size R under the condition that it has not collapsed to a black hole.

• Assume a roughly spherical geometry throughout. (Brane world geometry modifies the results substantially!)

• Dominant configurations are thermal[1], characterized by a temperature T, energy density T^D and entropy density T^{D-1}. The total energy and entropy of the region are

$$E \sim R^{D-1} T^D \ , \quad S \sim R^{D-1} T^{D-1} \quad . \tag{1}$$

Substituting into (1), we obtain the bound:

$$M_*^{2-D} R^{D-1} T^D \ < \ R^{D-3} \ , \tag{2}$$

which implies

$$T \ < \ \left(M_*^{D-2} R^{-2} \right)^{1/D} \quad , \tag{3}$$

and the entropy bound

$$S \sim R^{D-1} T^{D-1} \ < \ M_*^{(D-2)(D-1)/D} \ R^{D-1} \ R^{-2(D-1)/D} \quad , \tag{4}$$

or

$$S \ < \ R^{D-3+2/D} \ \sim \ R^{(D-2)+(2/D-1)} \quad . \tag{5}$$

[1]Temperature here is fictitious. It lets us characterize the dominant configurations in phase space, since in thermal equilibrium the entropy density is maximal for a given energy density.

• This is always at least as strong as the Holographic bound: $S < R^{D-2}$. The two bounds coincide when $D = 2$: boundary of the black hole is simply two points. For $D = 4$ we obtain 't Hooft's result that $S < A^{3/4}$. (Note: we have ignored black holes!)

Holographic Bound (HB): entropy of a spacelike region less than $A/4$.

• Strongest conjecture allows arbitrary choice of volume and boundary. (Note Susskind construction involves black hole formation, so boundary region is determined dynamically.) Support from string theory: AdS dualities.

• Covariant generalization [Bousso, 1999] involving lightsheets, necessary for dynamic spacetimes, reduces to the simple version in the examples considered here.

Brane Worlds: the Planck scale M_* is only of order TeV, so entropy per unit area is small. We can find systems in our universe which violate the entropy bounds:

1) early universe during nucleosynthesis

2) core of supernova

$\rightarrow M_* > 10^6$ TeV !

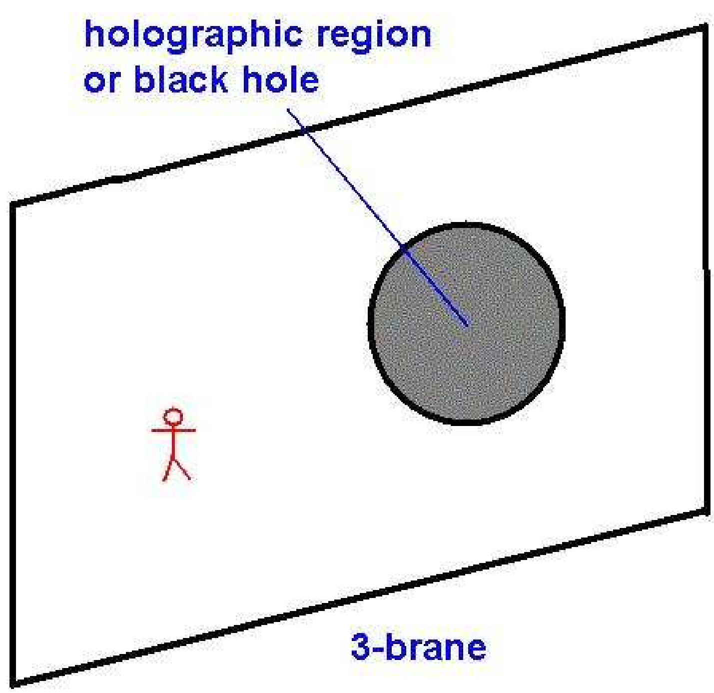

Figure 3: The brane world.

3 Dark Energy

Astrophysical data from WMAP and supernova surveys imply the existence of a cosmological constant-like component of the energy density of the universe.

(A) **Cosmology of Nonlinear Oscillations**, Phys.Lett.B567:9-11,2003, astro-ph/0305096.

New models of quintessence Q which do not require fine-tuned potentials. The Q field exhibits rapid oscillations rather than slow-rolling behavior today.

(B) Analysis of finite temperature effects on "tracker" models of quintessence. With PhD student Brian Murray.