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Recent Collaborators

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Publications (2005)

[82] **Thermal gravity, black holes and cosmological entropy**, with B. Murray, hep-th/0512033.

[81] **Message in the Sky**, with A. Zee, physics/0510102.

[80] **Entanglement entropy, black holes and holography**, with R. Buniy, hep-th/0510021, submitted to Physical Review Letters.

[79] **Is Hilbert space discrete?**, with R. Buniy and A. Zee, Phys.Lett.B630:68-72,2005, hep-th/0508039.

[78] **Minimum length from first principles**, with Xavier Calmet and Michael Graesser, hep-th/0505144. Honorable mention, 2005 Gravity Research Foundation competition.

Publications (2005) continued

[77] **Semi-classical wormholes and time machines are unstable**, with Roman V. Buniy, hep-th/0504003, to appear in Physics Letters B.

[76] **Instabilities and the Null Energy Condition**, with Roman V. Buniy, hep-th/0502203, to appear in Physics Letters B.

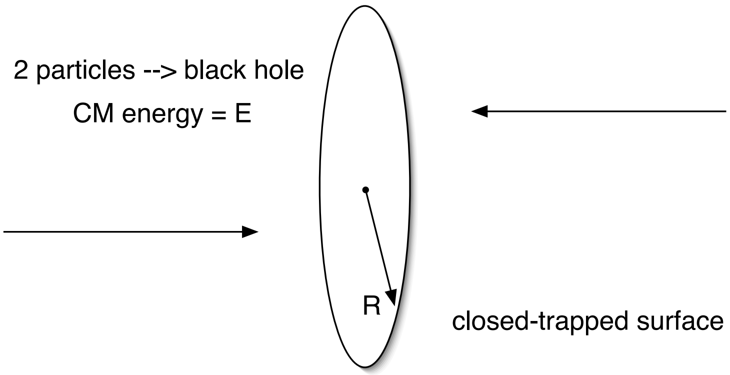
A recurring theme: gravitational collapse

Is there a general criteria for gravitational collapse leading to black hole formation?

Hoop Conjecture (K.S. Thorne): if energy E is confined in a region of size $R < E$, then collapse to a black hole is inevitable.

Schoen and Yau (1983), Eardley and Giddings (2002), Hsu (2002). Latter papers relevant to question of mini-black hole production at LHC.

Hoop conjecture



Minimum Length from GR and QM

X. Calmet, M. Graesser, S. Hsu (Caltech), PRL93:211101,2004

(1) Uncertainty principle: $\Delta p > \Delta x^{-1}$
(2) Hoop conjecture (gravitational collapse): $E < R$

Consider a particle probe with energy Δp. Then

$$R \sim \Delta x \; > \; \Delta p \; > \; \Delta x^{-1}$$

or

$$\Delta x > l_P$$

where

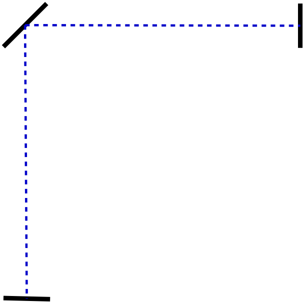
$$l_P^2 \sim G_N$$

A hard probe eventually becomes a black hole!

Minimum Length from GR and QM

What about Interferometers? An interferometer can measure very small changes in position without using short distance probes. e.g., LIGO has sub-femtometer resolution, using IR lasers (1000 nm): $\Delta x << \lambda$

Is there a device-independent minimum length bound?



Minimum Length from GR and QM

General theorem, using only QM and GR

Heisenberg picture: $\hat{x}(t) = \hat{x}(0) + \hat{p}(0)t/M$

Non-zero commutator: $[\hat{x}(t), \hat{x}(0)] = it/M$

Using Hoop Conjecture and Uncertainty Principle:

$$\Delta x(t)\Delta x(0) \; > \; t/M \; > \; R/M \; > \; 1$$

or, $\Delta x \; > \; l_P$!

No meaning to lengths less than Planck length – cannot measure, even in gedanken experiment. Does quantum gravity cut off its own UV divergences? Is spacetime discrete?

Is Hilbert space discrete?

Buniy, Hsu and Zee, Phys.Lett.B630:68-72,2005, hep-th/0508039.

...discretization of spacetime naturally suggests discretization of Hilbert space itself. Specifically, in a universe with a minimal length (for example, due to quantum gravity), no experiment can exclude the possibility that Hilbert space is discrete.

Hilbert space: space of $|\psi\rangle$'s spanned by basis vectors $|n\rangle$,

$$|\psi\rangle = \sum_{n=1}^{N} a_n |n\rangle, \tag{1}$$

modulo rescaling by an arbitrary complex parameter. If a_n are continuous complex parameters, Hilbert space is continuous even if spacetime is discrete.

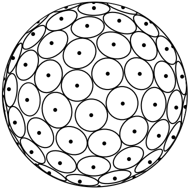
Is Hilbert space discrete?

Two arguments for discreteness. (More precisely: inability of experiments to exclude sufficiently small discreteness.)

1) Consider single qubit and Stern-Gerlach measuring device.

$$|\psi\rangle = \cos\theta\,|+\rangle + e^{i\phi}\sin\theta\,|-\rangle,$$

Spatial discreteness implies only a finite number of orientations for the device, or θ and ϕ effectively *discrete* parameters.



Result: discrete Bloch sphere; qubit specified by *finite* amount of classical information.

Is Hilbert space discrete?

2) Consider particle of energy E, interacting with an external probe of energy E', which measures the phase ϕ of its wavefunction.

Interaction takes place over a time interval Δt. Particle's phase necessarily evolves during the time interval, so $\Delta\phi \sim E\Delta t$. Causality requires $\Delta t > R$, where R is the size of the probe (or the portion of it which interacts with the particle).

$R > E'$, or the probe would have already collapsed into a black hole. Finally, using energy-time uncertainty, $E' > (\Delta t)^{-1}$,

$$\Delta\phi \sim E\Delta t > EE' > E^2/\Delta\phi, \tag{2}$$

or $\Delta\phi > E$. So a phase discreteness ϵ smaller than $\sim E$ (recall, we use Planck units) is undetectable experimentally.

A simple entropy bound

Buniy and Hsu, hep-th/0510021.

Black hole entropy bounds and AdS/CFT duality suggest that the number of degrees of freedom might be much smaller than originally suspected.

Ordinarily, we expect that the dimensionality of the Hilbert space describing a volume V is $\dim\mathcal{H} = c^V$ ($c = 2$ for qubits). But, there are indications that the entropy

$$S \sim \ln \{\text{\# of microstates } N\}$$

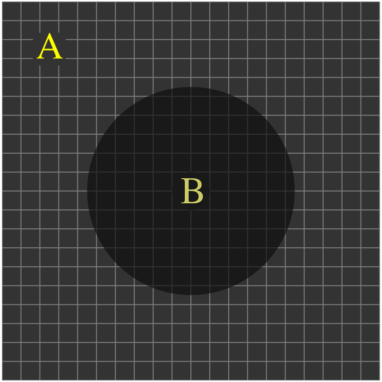
is bounded by the boundary area in Planck units:

$$S \;\sim\; \ln \dim\mathcal{H} \;<\; A\,.$$

"Holography" = the world is secretly 2+1 dimensional?

A simple entropy bound

Susskind construction.



Black hole entropy = $\frac{1}{4}$Area.

Second law of thermodynamics.

→ entropy of **any system** bounded by its area in Planck units.

A simple entropy bound

Is there a hard cutoff on quantum field theory modes imposed by holography?

This would lead to problems with locality, unitarity.

Simpler possibility: the size of Hilbert space is not limited by gravity, but the realizable set of states (pure or mixed) is limited by gravitational collapse. We show that for a density matrix ρ_B, describing a region B of size R, the condition

$$\mathrm{tr}(\rho_{\mathrm{B}} H_{\mathrm{B}}) = \langle H_{\mathrm{B}} \rangle < R.$$

is sufficient to guarantee

$$S = \mathrm{tr}\, \rho_B \ln \rho_B < A^{3/4} < A \,.$$