Thermal gravity, black holes and cosmological entropy

Brian Murray

ITS, University of Oregon
Adviser: Steve Hsu

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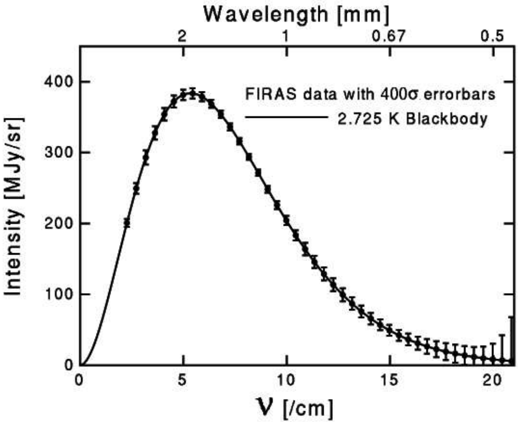
Motivation: the low entropy of our universe



The matter and radiation in the early universe was in a high entropy thermal state. But what about gravity?

Gravity is different because it is a long range, unscreened, universally attractive force. Systems subject to gravitational interactions maximize their entropy by clumping and undergoing gravitational collapse to form black holes.

Motivation: the low entropy of our universe

While matter fields were initially in a high entropy thermal state, either the gravitational degrees of freedom were not in a thermal state (Penrose '79) or they did not have a means by which to explore their phase space, i.e., they are not ergodic.

If the gravitational degrees of freedom were thermal and they did have a means to explore their phase space, spacetime would have evolved away from the smooth Friedmann-Robertson-Walker (FRW) universe to a very clumped, highly entropic configuration.

I will discuss a means by which gravity may explore its phase space if the gravitational degrees of freedom are ever thermal, either because of the initial conditions or due to interactions with thermal matter fields.

Planck units: $G_\mathrm{N} = c = \hbar = k_\mathrm{B} = 1$, e.g. $A_\mathrm{Pl} = G_\mathrm{N}\hbar/c^3 \simeq 10^{-66}\mathrm{cm}^2$

Statistical mechanics of thermal gravitons

Consider a box of thermal gravitons (i.e., fluctuations on the metric, gravitational waves) at temperature T.

The probability for a fluctuation to lead to a black hole of radius R and energy $E = R/2$ is

$$P(R) \sim N e^{-E/T},$$

where $N = e^S$ is the multiplicity of configurations with energy $E = R/2$, and S is the entropy of such a configuration.

Recall that the Helmholtz free energy is $F = E - TS$, then

$$P(R) \sim e^{-F/T}. \tag{1}$$

Statistical mechanics of thermal gravitons

We assume that black hole entropy is accounted for by gravitational microstates, as suggested by results from string theory (Strominger, Vafa ’96).

Using the Bekenstein (’73) Hawking (’75) formula for black hole entropy $S_{\mathrm{BH}} = A/4$, where $A = 4\pi R^2$, we see

$$F(R) = R/2 - \pi T R^2. \tag{2}$$

The maximum free energy is

$$F_* = (16\pi T)^{-1}, \tag{3}$$

and the critical radius or saddlepoint radius, which maximizes the free energy is

$$R_* = (4\pi T)^{-1}.$$

Statistical mechanics of thermal gravitons

Black holes radiate at a temperature $T_{\rm BH}$ related to their size:

$$T_{\rm BH}(R) = (4\pi R)^{-1}.$$

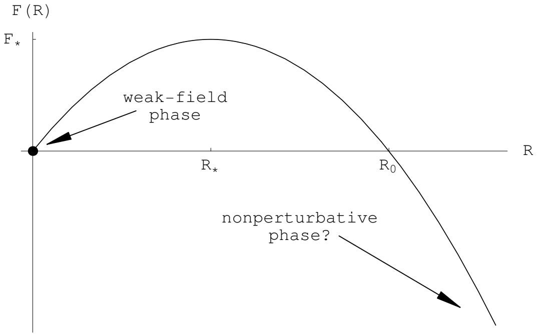
A black hole of critical size is the same temperature as the environment:

$$T_{\rm BH}(R_*) = T.$$

Black holes smaller than R_* are hotter than the environment.
They shrink to zero size via the Hawking effect, leaving behind a weak-field phase of a thermal population of gravitons.

Black holes larger than R_* are colder than the environment.
They grow without bound, possibly leading to a new nonperturbative phase of gravity, which is not asymptotically flat, and in which spacetime is all or partially filled with black holes.

Statistical mechanics of thermal gravitons



This nonperturbative phase is highly entropic and occupies an exponentially larger volume of phase space than the smooth weak-field phase.

In a radiation dominated FRW universe, the Hubble radius goes like $d_H \sim T^{-2}$. Again the critical radius is $R_* = (4\pi T)^{-1}$.

$d_H \gg R_* \gg 1$ so our analysis encounters no difficulties due to causality or quantum gravity.

Statistical mechanics of thermal gravitons

As in the usual case of a first order phase transition, nucleation of supercritical black holes is controlled by the free energy at the saddlepoint F_*. These supercritical black holes may be thought of as bubbles of the new highly entropic nonperturbative phase.

The nucleation rate per unit volume is

$$\lambda(T) \sim T^4 e^{-F_*/T}, \tag{4}$$

where again

$$F_* = (16\pi T)^{-1}.$$

Statistical mechanics of thermal gravitons

These results on the instability of hot, flat space were previously obtained by Euclidean path integral methods (Gross, et al. '82). In that approach the imaginary part of the Euclidean action is interpreted as corresponding to a decay rate.

By placing the system inside a cavity of finite size, York ('86) redefined the system in such a way that the partition function is finite, i.e., the canonical ensemble exists.

Our approach gets the basic physics right and relies only on knowing how the entropy of the system scales with size. This is similar to the work of Kapusta ('84).

Statistical mechanics of thermal gravitons

What if the gravitational degrees of freedom were not thermalized, but only matter degrees of freedom were hot?

Consider a box of thermal photons at temperature T.

The Hoop Conjecture (Thorne '72) states that if a fluctuation of size R and energy E satisfies $E > R/2$ it will evolve into a black hole.

Again the probability for a fluctuation with $E = R/2$ goes like

$$P(R) \sim N_\gamma e^{-E/T},$$

where $N_\gamma = e^S$ is the multiplicity of photon configurations with $E = R/2$, and S is the entropy of such a configuration.

Statistical mechanics of thermal gravitons

Use the entropy bound on a matter system ('t Hooft '93):

$$S < S_{\max} \sim A^{3/4}.$$

Recall that in thermal equilibrium $E \sim R^3 T^4$, and $S \sim R^3 T^3$. Further, require that the system has not yet undergone gravitational collapse, i.e., use the hoop conjecture: $E < R/2$.

Once a black hole has been nucleated, it evolves according to the previous analysis. Therefore, we want to know about the nucleation of black holes of critical size R_*.

Note that $A_* = 4\pi R_*^2 \gg 1$, and so nucleation of black holes of critical size in a photon gas is suppressed relative to the case of a graviton gas, by a factor of

$$\frac{N_\gamma}{N_g} \sim \exp\left(A_*^{3/4} - A_*/4\right) \sim \exp\left(-\frac{1}{16\pi T^2}\right) \ll 1.$$

Thermal matter can thermalize gravitons

General Relativity (GR) is not the quantum theory of gravity (it is nonrenormalizable, etc.).

However, at sufficiently low energies (roughly below M_{Pl}), it may consistently be treated as a low-energy effective field theory (EFT) (Donoghue '94).

An EFT (Weinberg '79) is an energy expansion, in which all terms consistent with the symmetries of the system are included in the effective Lagrangian. Terms with more derivatives are suppressed by a mass scale to the appropriate power (in this case, roughly M_{Pl}).

There is no problem in using an EFT even if one does not know the high energy completion of the model, which in the case of quantum gravity may be, e.g., string theory, etc.

Thermal matter can thermalize gravitons

Treating GR as a low-energy EFT, one can do a standard freeze-out calculation for the graviton.

Boltzmann heuristics motivate calculating the temperature at which the interaction rate of gravitons with matter fields is equal to the Hubble expansion rate: $\Gamma(T_{\mathrm{dec}}) = H(T_{\mathrm{dec}})$.

For scalar QED with N scalars, at tree level we find

$$T_{\mathrm{dec}} \simeq \frac{3}{5\alpha N^{1/2}},$$

where $\alpha = Q^2/4\pi$, and Q is the charge of the scalars.

We estimate the one-loop correction to be small relative to the tree-level term for $T < T_{\mathrm{pert}} \simeq 0.1$, independent of N.

Thermal matter can thermalize gravitons

There exists a large class of models for which an EFT calculation shows that gravitons are frequently interacting with thermal matter at temperatures well below the scale at which quantum corrections to gravity become important.

Therefore, even if the gravitational degrees of freedom are initially cold (i.e., not thermal), interactions with thermal matter can heat them up.

Once the gravitational degrees of freedom are thermal, we can ask whether they are able to explore phase space, in order to access highly entropic (i.e., very clumpy) configurations.

Percolation of high entropy black hole phase

Under what conditions does a phase transition to the high entropy black hole phase take place in an FRW universe?

Assume gravitons are in thermal equilibrium between times t_0 and t_1. The volume fraction in the weak-field phase is:

$$p(t_0, t_1) = \exp\left[-\int_{t_0}^{t_1} dt' V(t', t_1)\lambda(t')\right]. \tag{5}$$

In a flat FRW spacetime, the volume of a sphere expanding at constant speed b between times t' and t_1 is given by

$$V(t', t_1) = \frac{4\pi}{3}\left[a(t')b\int_{t'}^{t_1} \frac{dt''}{a(t'')}\right]^3,$$

where $a(t)$ is the scale factor at time t.

Percolation of high entropy black hole phase

Recall from earlier that the black hole nucleation rate per unit volume takes the form:

$$\lambda(T) \sim T^4 \exp\left(-\frac{1}{16\pi T^2}\right).$$

In a radiation dominated FRW universe, the scale factor goes like

$$a \sim t^{1/2},$$

and time and temperature are related by

$$t \simeq 0.3 g_*^{-1/2} T^{-2},$$

where g_* is the effective number of degrees of freedom.

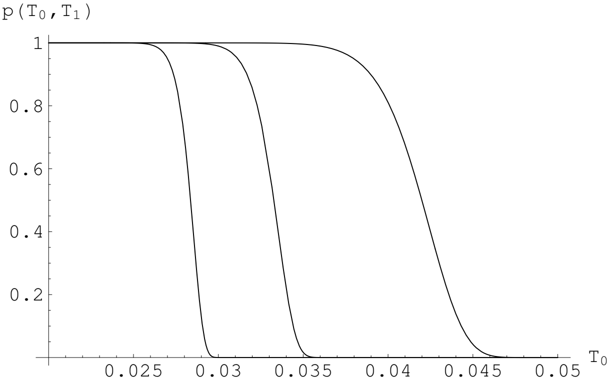
Therefore, the volume fraction in the weak-field phase can be rewritten as a function of temperature:

$$p(T_0, T_1) = \exp\left[-\frac{b^3}{g_*^2} f(T_0, T_1)\right]. \qquad (6)$$

Percolation of high entropy black hole phase

Define a critical initial temperature $T_0^c = T_0^c(T_1, g_*, b)$.

The phase transition does not occur for $T_0 < T_0^c$.



Volume fraction in the weak-field phase for $T_1 = 10^{-6}$, $g_* = 10^2$ and $b = 10^{-1}, 10^{-2}, 10^{-3}$ (left to right).

Increasing T_1, g_* or decreasing b causes T_0^c to increase logarithmically.

Percolation of high entropy black hole phase

Independent of the parameters in the percolation model (T_1, g_*, b), the critical initial temperature is of the same order as the scale at which gravity becomes perturbative:

$$T_0^c \sim T_{\rm pert} \sim 10^{-2} - 10^{-1}.$$

Therefore, a phase transition to the high entropy black hole phase can be avoided, *if low entropy weak-field initial conditions are set at a scale no higher than the scale at which gravity becomes perturbative.*

In this sense our low entropy, weak-field universe is not fine tuned, even though weak-field initial conditions may represent a set of measure zero in the space of possible initial conditions.

Conclusions

We examined a possible first order phase transition of spacetime to a black hole phase with high entropy.

Percolation of the high entropy phase occurs if gravitons are ever in a thermal state with temperature above T_0^c, either because they were born hot at the Planck epoch or because they were thermalized due to interactions with thermal matter.

Hot gravitons with temperature slightly below T_0^c will not lead to a phase transition; they will simply be red-shifted away.

The weak-field phase may represent a set of measure zero, but because T_0^c is of the same order as the energy scale below which quantum gravity effects are small, once weak-field initial conditions have been chosen no additional fine tuning is required to avoid a phase transition to the black hole phase.