Black Hole Production from Particle Collisions

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Hoop Conjecture and Black Holes

First proposed by Kip Thorne at Caltech, based on intuition from (mildly) relativistic astrophysics.

Conjecture: Given a blob of stuff with total energy E, let

$$R_s \;=\; G_N E \;=\; E/M_{\rm Planck}^2 \quad .$$

If one can pass a hoop of radius R_s around the blob, then it will eventually evolve into a black hole!

Recall that, in general relativity one often finds solutions with special behavior at $GM \sim R$.

Schwarzschild metric:

$$ds^2 = (1 - 2GM/r)dt^2 - (1 - 2GM/r)^{-1}dr^2 - r^2 d\Omega^2 \quad .$$

Does this apply to highly relativistic situations where almost all of the energy is kinetic? e.g., two colliding massless particles:

Here b = impact parameter. For R_s to be larger than the Planck length ($L_{\rm Planck} = 10^{-33}$ cm), must have $E >> M_{\rm Planck} \sim 10^{19}$ GeV.

Impossible with current or imaginable collider technology!

But, what if gravity is much stronger than we thought?!?

Large Extra Dimensions

Recently (Arkani-Hamed et al., 1998), it was pointed out that we might live on a 4 dimensional submanifold of a higher-dimensional space:

If so, the strength of gravity which we perceive is affected by the extent of the extra dimensions:

$$F_{4+d}(r) \ \sim \ G_{4+d}\frac{m_1 m_2}{r^{2+d}} \sim \frac{G_{4+d}}{R^d}\frac{m_1 m_2}{r^2}$$

where $[G_{4+d}] = L^{2+d}$.

For $d = 2$, if we choose $R \sim 10^{-3}$ cm, then $L \sim \mathrm{TeV}^{-1}$. So, a six dimensional universe (in which we live on a 4d slice) with 10^{-3} cm extra dimensions has a Planck scale which is similar to the electroweak scale!

No "hierarchy problem"!

In this universe, TeV-scale collisions would probe quantum gravity. What would we see?

Phenomenology of black hole creation

If the hoop conjecture applies, black hole production will dominate other cross sections at energies larger than M_{Planck}.

Suppose we make small black holes in colliders or cosmic ray collisions.

Most black holes produced will have large angular momentum ($b \neq 0$).

Multiple stage evolution: first, radiate away angular momentum, then Hawking evaporate.

Reminder: Hawking radiation due to quantum fluctuations near horizon. Hawking temperature: $T \sim M^2_{\text{Planck}}/M$

Signatures are very striking. Will we see them at LHC?

If black hole creation is dominant process at high energies, we will lose the ability to resolve length scales less than the Planck length! **The end of high energy physics?**

What is the cross section for black hole production?

Analysis using *classical* general relativity: Eardley and Giddings (ITP, Santa Barbara)

First, some machinery.

Closed Trapped Surfaces and the Hawking-Penrose Theorem

How does one define (rigorously) what a black hole is? **Closed Trapped Surface S** = closed, compact, spacelike surface with convergent null normals. i.e., light rays emanating from **S** converge rather than diverge.

Hawking-Penrose theorem: modulo some conditions on the Stress-Energy tensor $T_{\mu\nu}$, the formation of a CTS implies a singularity in the future evolution of the Einstein equations.

Cosmic Censorship Conjecture: there are no "naked" singularities. All singularities are hidden from an observer at infinity by an event horizon. In other words, singularities in the evolution of the Einstein equations imply black hole formation.

Eardley-Giddings (EG) Construction

Aichelburg-Sexl metric and impulsive waves: metric for highly boosted massless particle has been known for some time. Start with Schwarzschild metric, apply boost γ, take limit of large γ and small mass carefully!

$$ds^2 = -du dv + H_{ij} H_{jk} dx^i dx^j$$

$$H_{ij} = \delta_{ij} + \frac{1}{2} \nabla_i \nabla_j \phi(\mathbf{x}) u \theta(u)$$

u and v are related to lightcone coordinates $t \pm z$. Here $\nabla^2 \phi = -16\pi R_s \, \delta^{2+d}(\mathbf{x})$. Note $\phi(\mathbf{x})$ is singular near $|\mathbf{x}| = 0$: shockwave!

Superimpose two A-S metrics, one coming from $+z$ and the other from $-z$.

EG are able to construct a CTS $\mathbf{S}$ in the part of the spacetime diagram where the two shockwaves have not yet had time to interact. In 4 dimensions, their construction works as long as $b < 1.6 R_s$, so they obtain a bound: $\sigma > \pi b^2 \sim 8 R_s^2$. This geometric cross section grows with energy: $\sigma > 8 G^2 E^2$.

Define $\mathbf{S} = \mathbf{S_1} \cup \mathbf{S_2}$, with $\mathbf{S_1} : u = 0, v = -\psi_1(\mathbf{x}) \leq 0$ and $\mathbf{S_2} : v = 0, u = -\psi_2(\mathbf{x}) \leq 0$. Then, marginal CTS condition is:

$$\nabla^2 \psi_{1,2} = \phi(\mathbf{x})$$

(plus continuity conditions) inside some curve $\mathbf{C}$ on the intersection surface $u = v = 0$.

Quantum Production of Black Holes

S. Hsu, University of Oregon, hep-ph/0203154.

An S-matrix for gravity.

$$S_{fi} \ = \ \langle f|\hat{\mathbf{S}}|i\rangle \ = \ \lim_{T\to\infty} \ \langle f| \ e^{-iHT} \ |i\rangle$$

What are states $|i\rangle, |f\rangle$?

Penrose diagram:

Black hole quantum numbers: mass, angular momentum, charge.

$$|\mathbf{M},\mathbf{J},\mathbf{Q}\rangle = \mathbf{a}^{\dagger}_{\mathbf{M},\mathbf{J},\mathbf{Q}}|\mathbf{0}\rangle$$

But, our black holes are semiclassical since $\mathbf{M} >> M_{\rm Planck}$.

Path integral expression for the S-matrix

$$\langle f|S|i\rangle \;=\; \int d\phi_i \; d\phi_f \; D\phi \; \Psi_i[\phi(T_i)] \; \Psi_f[\phi(T_f)] \; e^{iS[\phi]} \quad . \tag{1}$$

For two particles $\rightarrow$ semi-classical final state:

$$S\,[\mathbf{b}^*, 2] \;=\; \lim_{T_i, T_f \rightarrow \mp\infty} \int d\phi_f \, d\phi_i \, D\phi \; e^{\Gamma} \; , \tag{2}$$

where the effective action Γ is

$$\Gamma\;[\,\phi\,] \;=\; \ln \alpha_R \cdot \phi_i \;\alpha_L \cdot \phi_i \;+\; B_i\,[0, \phi_i] \;+\; iS\;[\,\phi\,] \;+\; B_f\,[b^*, \phi_f] \;. \tag{3}$$

Boundary value problem for semiclassical trajectories

Vary wrt $\phi(x)$ for $T_i < t < T_f$, to get usual field equations:

$$\frac{\delta S}{\delta\phi(x)} \;=\; 0 \;. \tag{4}$$

Varying wrt boundary field $\phi_i(k)$, gives

$$i\,\dot{\phi}_i(\vec{k}) \;+\; \omega_k \phi_i(\vec{k}) \;=\; \sqrt{2\,\omega_{\mathbf{k}}}\;\left(\frac{\alpha_R(\vec{k})}{\alpha_R \cdot \phi_i} \;+\; \frac{\alpha_L(\vec{k})}{\alpha_L \cdot \phi_i} \right)\; e^{-i\omega_k T_i} \;. \tag{5}$$

In the asymptotic region $t = T_i \rightarrow -\infty$, get plane wave superposition

$$\phi_i(\vec{k}) \;=\; \frac{1}{\sqrt{2\,\omega_{\mathbf{k}}}}\;\left(u_{\mathbf{k}}\; e^{-i\omega_k T_i} \;+\; u^*_{-\mathbf{k}}\; e^{i\omega_k T_i} \right) \;. \tag{6}$$

Equation (5) then reduces to the requirement

$$u_{\mathbf{k}} \;=\; \frac{\alpha_R(\vec{k}) \;+\; \alpha_L(\vec{k})}{\left(1 \;+\; \int d^3k \;\alpha_R(\vec{k})\;\alpha_L(\vec{k}) \right)^{1/2}} \;, \tag{7}$$

using the normalization condition on $\alpha_{L,R}$.

We obtain a direct relation between classical solutions satisfying our boundary value problem, and quantum amplitudes.

Quantum Corrections and a scaling limit

The semiclassical approximation will be accurate as long as quantum corrections (due to small fluctuations about the classical solution) are small.

Gravity as an effective theory: in GR quantum corrections generate higher powers of curvature:

$$\mathcal{L}_{\text{eff}} = \sqrt{g}\,\left[\; c_0 + c_1 R + c_2 R^2 + c_3 R_{\mu\nu} R^{\mu\nu} + c_4 R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + \cdots \;\right]$$

Coefficients c_i are $\mathcal{O}(M_{\text{Planck}}^{-k})$. We get an expansion in powers of curvature over the Planck scale.

Quantum corrections negligible if curvature is small.

Can also view as initial value problem. Classical Einstein equations arise from leading term in $\mathcal{L}_{\text{eff}}$. Corrections neglected as long as curvature of solution is small.

If large curvature builds up, suddenly can't neglect higher order terms: quantum evolution of system might differ from classical evolution.

Curvature and Aichelbug-Sexl metrics: the pure A-S metric has infinite curvature, but can be "smoothed" to a metric with curvature **R** (in Planck units):

$$\mathbf{R} \sim (R_s/r)^2 (L_{\rm Planck}/r)^2 \quad ,$$

where r is the smoothing scale. By taking $r >> L_{\rm Planck}$ while keeping $R_s >> r$, we can keep the curvature small everywhere and still form a black hole!

Remember, we can take $R_s \sim G_N E$ as large as we want by taking $E \to \infty$.

This scaling limit corresponds to "colliding jupiters"

Kohlprath and Veneziano, gr-qc/0203093, have verified that the Eardley-Giddings construction can be generalized to these smoothed metrics.

Modulo some pathology of quantum gravity, particle collisions should behave like the scaling limit – only modify metric at short distances $\sim r$**, while horizon formation is at large distances** $\sim R_s$**.**

Conclusions

If extra dimensions solve the hierarchy problem, we may see very dramatic things in the next generation of colliders.

Black hole production amplitudes can be computed reliably as long as $E >> M_{\rm Planck}$, and yield geometrical (large!) cross sections. *Gravity is Special!*

The existence of a limiting fundamental length scale $L_{\rm Planck}$ can be deduced from semiclassical considerations!

Under further investigation:

Analytic structure of scattering amplitudes at trans-Planckian energies: virtual black holes and radiative electroweak corrections.

Singularities in the semiclassical phase: gravitational action as a surface integral over Bondi masses.

Implications for string theory?