Dark Energy and the Future of the Universe

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Cosmological models

Einstein equations from general relativity:

$$G_{\mu\nu} = 8\pi G(T_{\mu\nu} + \Lambda g_{\mu\nu}).$$

$G_{\mu\nu}$ is the Einstein tensor (built from the metric $g_{\mu\nu}$ and its derivatives),

G is Newton's constant,

$T_{\mu\nu}$ is the energy-momentum tensor of the non-gravitational matter fields.

Λ is the mysterious cosmological constant, to be discussed later.

Cosmological models

The Friedman-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - R(t)^2 \left(dr^2 + r^2 d\Omega^2\right)$$

describes a homogeneous, isotropic universe. (We've assumed a flat universe, with $k = 0$.) In the cosmological frame

$$T_{\mu\nu} = \mathrm{diag}\left(\rho(t), p(t), p(t), p(t)\right)$$

Cosmological models

The Einstein equations ($\Lambda = 0$) become

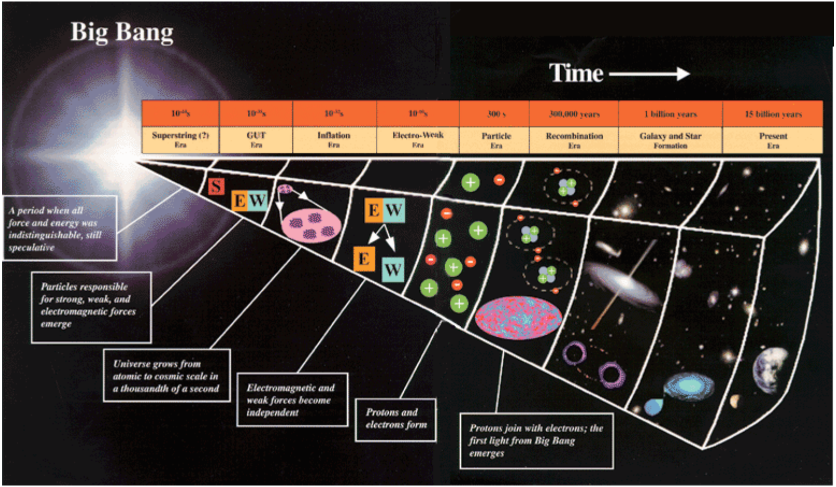
$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}\rho$$

plus an energy conservation relation between p and ρ:

$$d(\rho R^3) = -p d(R^3) \ .$$

Given the equation of state of the matter (p and ρ), we can determine the large scale evolution of the universe.

History of the universe



Comments

The FRW big bang model successfully predicts nucleosynthesis (light element abundances), the cosmic microwave background and its temperature fluctuations.

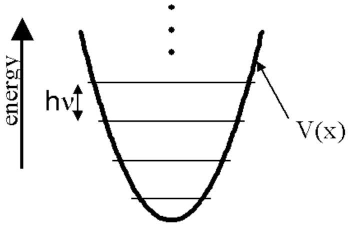
The universe seems to be very close to flat ($k = 0$).

However, there is still that mysterious Λ parameter...

Cosmological constant

Einstein's blunder? Λ is a kind of vacuum energy.

Arises from quantum corrections to $T_{\mu\nu}$. Each field theory mode $\phi(\vec{k})$ contributes a zero point energy $\omega(\vec{k}) \sim |\vec{k}|$ to Λ.



Quantum contributions to Λ are infinite, unless we cut off the momentum modes $\vec{k}$ at some scale M.

Even so, the natural size of $\Lambda \sim M^4$ – determined by short-distance physics.

Cosmological Constant

Λ behaves as matter with the peculiar equation of state

$$p = -\rho = -\Lambda$$

(see rhs of Einstein equation $= 8\pi G(T_{\mu\nu} + \Lambda g_{\mu\nu})$).

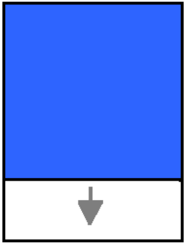
Negative pressure!

What if Λ dominates the energy-momentum from ordinary matter? Rapid expansion!

Negative work pdV done by expanding universe exactly enough to create a volume dV with energy density Λ. The process is self-sustaining!

Inflationary cosmology: $R(t) \sim e^{\sqrt{\Lambda}t}$. deSitter spacetime.

Cosmological Constant



The weirdness of negative pressure.

For ordinary equations of state, the energy density falls as the expansion does work on the piston – the early universe gets colder and less dense as it expands.

But, $p = -\rho = -\Lambda$ is a solution of the energy conservation equation $d(\rho R^3) = -pd(R^3)$. The energy density and pressure remain constant as the universe expands.

Cosmological Constant

The large scale structure of the universe appears to be determined by the details of physics at the shortest scales!

Dark Energy

Observations, including of Type Ia supernovae (SNe Ia) and Cosmic Microwave Background (CMB) anisotropy, imply that the expansion rate is increasing: $\ddot{R} > 0$.

The Einstein equations can be rewritten

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i).$$

In order for $\ddot{R} > 0$, the dominant component must have $p_{\rm de} < -\frac{1}{3}\rho_{\rm de}$ or *equation of state*

$$w_{\rm de} \equiv p_{\rm de}/\rho_{\rm de} < -1/3.$$

Any such component is called *dark energy* (including Λ, for which $w_\Lambda = -1$).

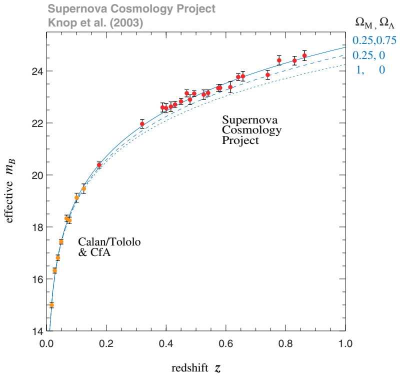
Observations: Type Ia Supernovae

SNe Ia are standardizable candles.

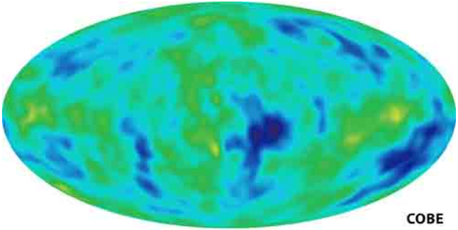
By measuring the spectra and light curves of a large number of SNe Ia, one can determine $R(t)$ and infer information about Ω_M and Ω_Λ.

($\Omega_i \equiv \rho_i/\rho_c$, where $\rho_c = 3H^2/8\pi G \sim 10^{-5}\,\mathrm{GeV\,cm^{-3}}$ is the critical density.)

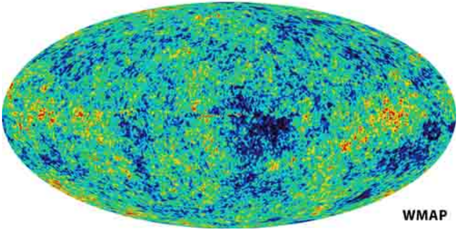
Conclusion: dark energy causes distant SNe Ia to be dimmer than they would be if there were only matter and radiation.



Observations: CMB Anisotropy



Launched in 1989 COBE (Cosmic Background Explorer) found small anisotropies in the temperature of the $T_{\text{CMB}} \simeq 2.725$ K Cosmic Microwave Background radiation.



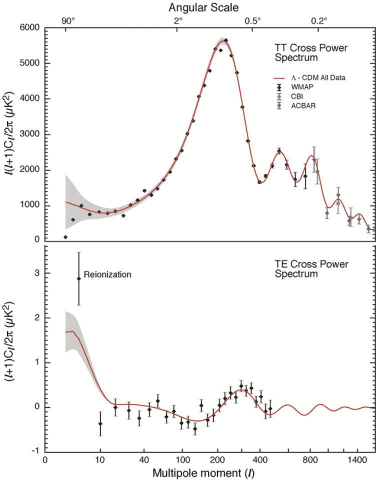
Launched in 2001 WMAP (Wilkinson Microwave Anisotropy Probe) had 45× the sensitivity and 33× the angular resolution of COBE.

Observations: CMB Anisotropy

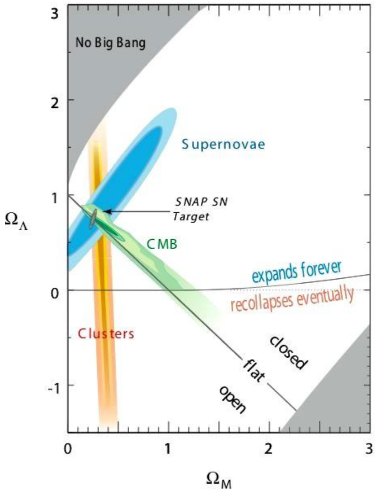
Combined CMB anisotropy data from WMAP and ground-based telescopes agree very well with the "standard" ΛCDM cosmological model.

The position of the first acoustic peak tightly constrains the geometry (curvature) of the universe ($k \simeq 0$).

Additional information can be gained from the polarization spectrum.



Observations: Future Prospects



Combined with galaxy cluster data, SNe Ia and CMB anisotropy observations tightly constrain universe contents.

It appears we are stuck with some form of dark energy.

Future CMB probes (e.g., Planck to be launched in 2007) and supernova surveys (SNAP - Supernova/Acceleration Probe still in planning stage) will allow us to go beyond the ΛCDM model.

Possibilities: dark energy is not Λ, i.e., $w \neq -1$, time-varying w, etc.

Why is Λ so small? And yet non-zero?

Theorists long believed that Λ would be exactly zero for some magical reason. Instead, we have this crazy Dark Energy!

Now that observations tell us it is non-zero, we struggle to understand why it is non-zero and yet so small:

$$\Lambda_{\rm obs}^{1/4} \sim 10^{-3}{\rm eV}.$$

A new fundamental scale of physics?

Why is Λ so small? And yet non-zero?

Weinberg's anthropic argument (1987)

Suppose $\exists$ many universes with different cosmological constants. (The string theory Landscape?) How likely is our value Λ_{obs}, given that life exists?

Assume that structure formation (galaxies, stars, etc.) is necessary for life. (Otherwise, uniform soup of particles!)

For $\Lambda > 200\,\Lambda_{\text{obs}}$, the universe becomes Λ-dominated before density perturbations can grow, and hence no galaxies form.

Perhaps this 'explains' the value of Λ?

Why is Λ so small? And yet non-zero?

Suppose that we fix all other parameters and vary only Λ. A flat prior-probability distribution is plausible, since Λ is determined by short-distance physics, and the range of viable values is quite narrow.

$$P(\Lambda)|_{\Lambda \sim \Lambda_{\text{obs}}} \sim \text{constant} \sim P(\Lambda = 0) + \mathcal{O}(\Lambda/M^4)$$

Result: In a Bayesian sense, Λ_{obs} is about 10% probable!

$$P(\Lambda < \Lambda_{\text{obs}} \cap \text{us}) = \int_0^{\Lambda_{\text{obs}}} d\Lambda\, P(\text{us}|\Lambda) P(\Lambda),$$

and assume $P(\text{us}|\Lambda) \propto$ baryon fraction in galaxies.

Why is Λ so small? And yet non-zero?

A prediction of the observed cosmological constant?

Alas, no. Weinberg's result assumes that all other parameters are held fixed. If one also varies (for example) the amplitude of primordial density perturbations (arising, e.g., from inflation), the probability of Λ_{obs} is reduced substantially to values as low as 10^{-4} (Graesser, Hsu, Jenkins and Wise, 2004).

Other possibilities

Perhaps Λ really is zero (for some unknown reason), and the dark energy is due to some dynamical field Q, often called 'Quintessence'.

One can build such models, but they are typically fine-tuned and implausible.

All we can say for now is that dark energy is a mystery. Perhaps the greatest discovery of all time?

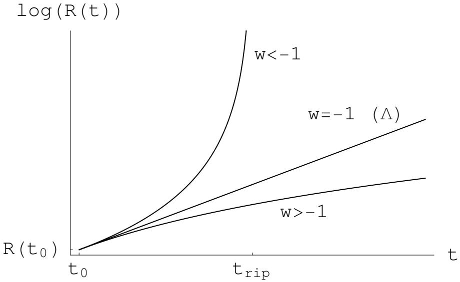
theorists: 0 observers: 1

The Big Rip

The observational data slightly favors $w = p/\rho < -1$.

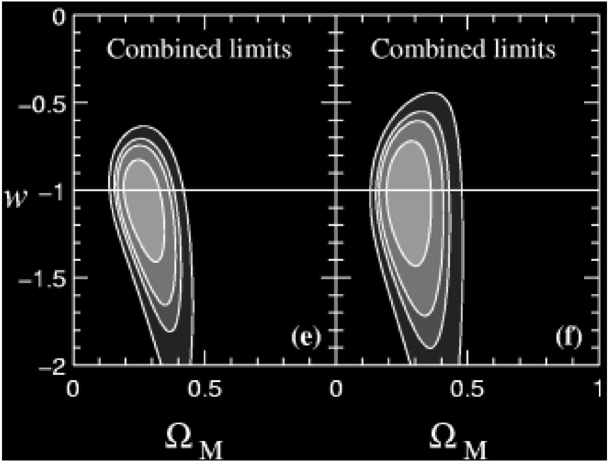
This requires that the pressure is more negative than $-\rho$.

More negative work pdV is done by the pressure than required to create additional volume with energy ρdV. The excess energy goes into accleration $\ddot{R}(t)$.



After a finite cosmological time, the universe hits a Big Rip singularity, with infinite acceleration at each point in space!

$w < -1$?



current bounds:
$-1.62 < w < -0.74$
(at 95% CL)

R. A. Knop *et al.*, Astrophys. J. **598**, 102 (2003) [astro-ph/0309368]

$w < -1$, NEC, instability references

NEC and stability:

R. Buniy and S. Hsu, Phys. Lett. B **632**, 543 (2006) [hep-th/0502203]

Wormholes and time machines:

R. Buniy and S. Hsu, Phys. Lett. B **632**, 127 (2006) [hep-th/0504003]

Earlier work on a scalar model instability:

S. Hsu, A. Jenkins and M. B. Wise, Phys. Lett. B **597**, 270 (2004) [astro-ph/0406043]

Dark energy

$\rho > 0$: to have enough matter to make Universe flat

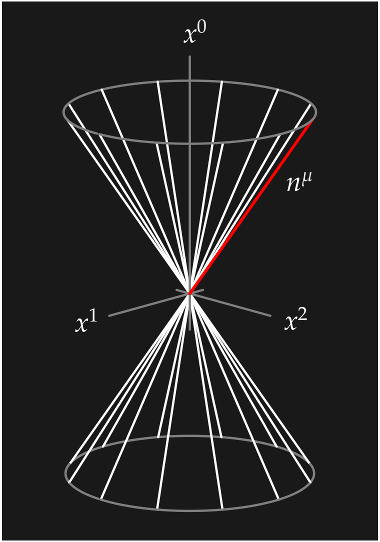
$p < 0$: to explain the observed acceleration

How negative can $w = p/\rho$ be?

$w_i = T_{00}/T_{ii}$: not Lorentz-invariant

$\rho + p_i \geq 0$: from Lorentz-invariant null energy condition

NEC

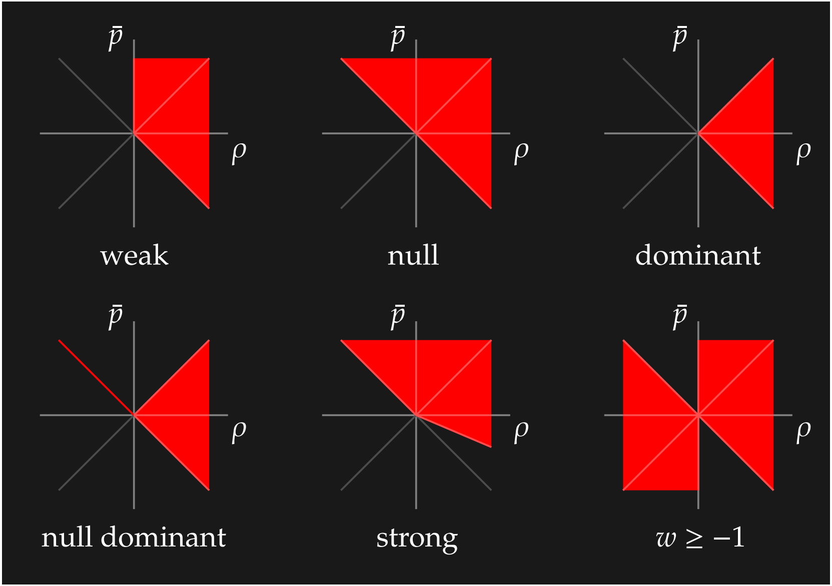


NEC: $T_{\mu\nu} n^\mu n^\nu \geq 0$

for null n^μ $(g_{\mu\nu} n^\mu n^\nu = 0)$

The energy density measured by an observer with the velocity $v^\mu = n^\mu$ is non-negative.

Energy Conditions and $w \geq -1$



Perfect fluid

$$l_{\text{typical}} \gg l_{\text{mean}}$$

Current: $j^\mu = J u^\mu \quad (u_\mu u^\mu = 1)$

Energy-momentum: $T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}$

Invariant: $J = (j_\mu j^\mu)^{\frac{1}{2}}$
(matter density in the rest frame)

Energy: $\rho = \rho(J)$

Pressure: $p = J\rho' - \rho$

Generality: arbitrary equation of state given by $\rho(J)$
(example: $\rho(J) = J$ for free fluid)

NEC: $T_{\mu\nu} n^\mu n^\nu = 2\rho' \underbrace{(u_\mu n^\mu)(u_\nu n^\nu)}_{\geq 0} \geq 0$

The NEC requires $\rho' \geq 0$.

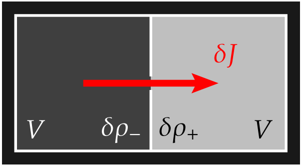
Clumping instability

Speed of sound: $s = (dp/d\rho)^{\frac{1}{2}} = (J\rho''/\rho')^{\frac{1}{2}}$

Real s: no exponentially-growing modes

If NEC is violated ($\rho'(J) < 0$) $\Rightarrow$ $\rho''(J) < 0$

What happens to fluid when $\rho'(J) < 0$ and $\rho''(J) < 0$?



$$\delta\rho_{\pm}(J) = \pm\rho'(J)\delta J + \tfrac{1}{2}\rho''(J)(\delta J)^2$$

$$\delta\rho(J) = \tfrac{1}{2}\rho''(J)(\delta J)^2 < 0$$

Clumping is energetically favorable.

Perfect fluid is stable only if the null energy condition is satisfied.

Classical field theories

Background: space-time with fixed metric $g_{\mu\nu}$

Variables: scalar fields ϕ_a and gauge fields $A_{a\mu}$

$$D_\mu \phi_a = \phi_{a,\mu} + C_a{}^{bc} A_{b\mu} \phi_c$$

$$F_{a\mu\nu} = A_{a\nu;\mu} - A_{a\mu;\nu} + C_a{}^{bc} A_{b\mu} A_{c\nu}$$

Action: $$S = \int d^d x \, |g|^{\frac{1}{2}} \left[\mathcal{L}(\phi_a, D_\mu \phi_a, F_{a\mu\nu}) + \tfrac{1}{2} f(\phi_a) R \right]$$

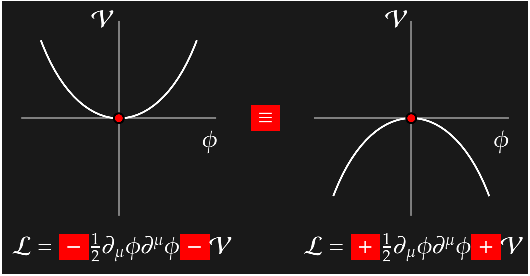
$\mathcal{L}$: an arbitrary Lorentz invariant function

f: an arbitrary function
($f = 1 - \frac{1}{2} \sum_a \xi_a \phi_a^2$ for non-minimal coupling)

R: Ricci scalar

Simple example

A scalar field with the opposite sign kinetic energy:



$\mathcal{V} = \frac{1}{2}m\phi^2$

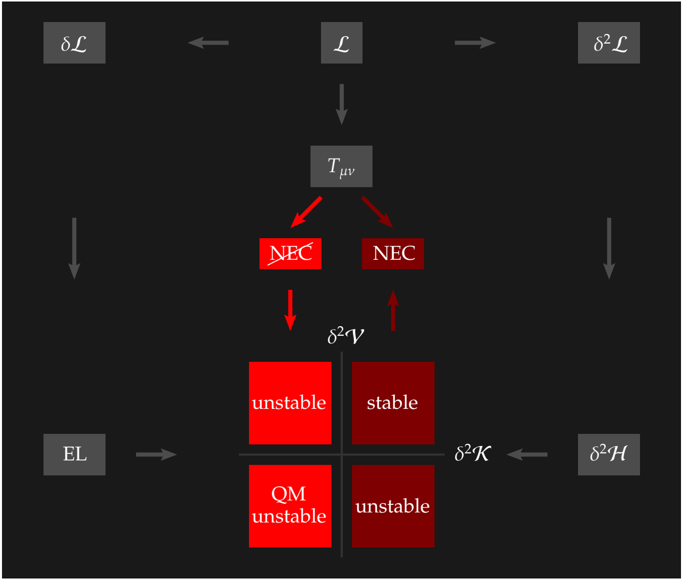
$\phi \propto e^{ik_\mu x^\mu}$

Instability: $k_\mu \in \mathbb{C}$

Energy-momentum: $T_{\mu\nu} = - \partial_\mu\phi\partial_\nu\phi - \mathcal{L}g_{\mu\nu}$

NEC: $T_{\mu\nu}n^\mu n^\nu = -\left(n^\mu\partial_\mu\phi\right)^2 < 0$

Strategy



$\delta\mathcal{L}$
$\mathcal{L}$
$\delta^2\mathcal{L}$
$T_{\mu\nu}$
NEC
NEC
$\delta^2\mathcal{V}$
unstable
stable
EL
$\delta^2\mathcal{K}$
$\delta^2\mathcal{H}$
QM
unstable
unstable

Stability

Theorem

For the theory given by the action

$$S = \int d^d x \, |g|^{\frac{1}{2}} \left[\mathcal{L}(\phi_a, D_\mu \phi_a, F_{a\mu\nu}) + \tfrac{1}{2} f(\phi_a) R \right],$$

only solutions satisfying the null energy condition can be stable.

Fermions

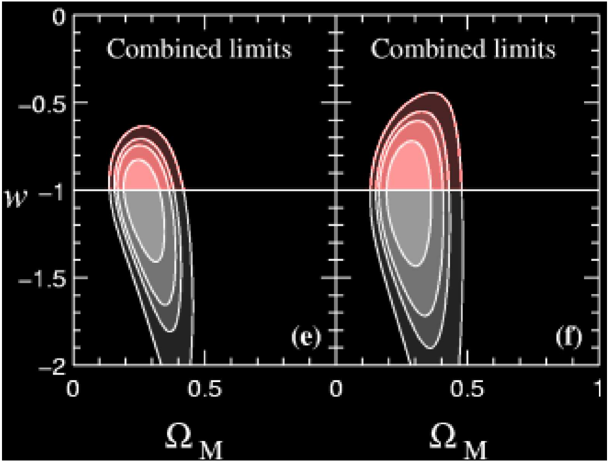
The bosonic part as before: $\mathcal{L}^{(\mathrm{b})} = \mathcal{L}(\phi_a, D_\mu \phi_a, F_{a\mu\nu}) + \frac{1}{2} f(\phi_a) R$

Add fermions: $\mathcal{L}^{(\mathrm{f})} = \bar{\psi} \left[i \not{D} - m(\phi) \right] \psi$

Conclusion

If the system with $\mathcal{L}^{(b)} + \mathcal{L}^{(f)}$ does not satisfy the NEC, the bosonic degrees of freedom are unstable.

$w \geq -1$



current bounds
and stability:
$-1 \leq w < -0.74$

R. A. Knop *et al.*, Astrophys. J. **598**, 102 (2003) [astro-ph/0309368]

Dark Energy

The greatest discovery of all time?

The single largest component (by energy) of the current and future universe.

Determines the large time evolution of the universe.

What is it?