DOE Research Report

Roman V. Buniy

ITS, University of Oregon

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Collaborators

- Stephen D. H. Hsu, UO
- Brian M. Murray, UO
- Thomas W. Kephart, Vanderbilt University
- Arjun Berera, University of Edinburgh, UK
- Eric J. Rawdon, Duquesne University
- Michael Piatek, University of Washington

Research topics

- Entanglement entropy (Oregon)
  - holography
  - Hawking radiation
  - information loss paradox
- Instabilities and the null energy condition (Oregon)
  - field theories in curved spacetimes
  - dark energy
  - wormholes and time machines
- Eccentric inflation (Vanderbilt, Edinburgh)
  - exact solutions
  - perturbations
  - data analysis (WMAP, etc.)
- Glueballs and knots (Vanderbilt, Duquesne, Washington)
  - numerical knot simulations
  - data analysis (PDG)

Published papers

- *"Instabilities and the null energy condition,"* with S. Hsu, Phys. Lett. B **632**, 543 (2006) [arXiv:hep-th/0502203]
- *"Semi-classical wormholes and time machines are unstable,"* with S. Hsu, Phys. Lett. B **632**, 127 (2006) [arXiv:hep-th/0504003]
- *"Is Hilbert space discrete?"* with S. Hsu and A. Zee, Phys. Lett. B **630**, 68 (2005) [arXiv:hep-th/0508039]
- *"Glueballs and the universal energy spectrum of tight knots and links,"* with T. W. Kephart, Int. J. of Mod. Phys. A **20**, 1252 (2005) [arXiv:hep-ph/0408027]
- *"Eccentric inflation and WMAP,"* Int. J. of Mod. Phys. A **20**, 1095 (2005) [arXiv:hep-ph/0408026]

Book chapter

- *"Universal energy spectrum of tight knots and links in physics,"* with T. W. Kephart, in *"Physical and numerical models in knot theory including applications to the life sciences,"* eds. J. Calvo, K. Millet, E. Rawdon and A. Stasiak, p. 45, World Scientific, Singapore, 2005 [arXiv:hep-ph/0408025]

Submitted manuscripts

- *"Asymmetric inflation: exact solutions,"* with A. Berera and T. W. Kephart [arXiv:hep-th/0511115]
- *"Entanglement entropy, black holes and holography,"* with S. Hsu [arXiv:hep-th/0510021]

Manuscripts in preparation

- *"The null energy condition and instability,"* with S. Hsu and B. M. Murray
- *"Asymmetric inflation: perturbations,"* with T. W. Kephart
- *"Tight links and their applications,"* with T. W. Kephart, M. Piatek, and E. Rawdon
- *"Bounces in field theory and cosmology: approximate solutions,"* with T. W. Kephart

Conference talks

- *“Entanglement entropy, black holes and holography,”* Miami Conference, December 2005
- *“Wormholes, dark energy, and the null energy condition,”* Frontiers in Contemporary Physics, Vanderbilt University, Nashville, May 2005
- *“Semi-classical wormholes and time machines are unstable,”* 21st Pacific Coast Gravity Meeting, Eugene, March 2005

Seminars

- Institute for Advanced Study, December 2005
- Los Alamos National Laboratory, November 2005 (2 seminars)
- University of California at Davis, October 2005
- University of Minnesota, October 2005
- Case Western Reserve University, October 2005
- University of California at Los Angeles, February 2005
- California Institute of Technology, February 2005
- University of California at Berkeley, February 2005

News reports

- *"Wormhole 'no use' for time travel,"* BBC, May 23rd, 2005, http://news.bbc.co.uk/2/hi/science/nature/4564477.stm
- *"Wormhole wanderers face a deadly dilemma,"* New Scientist, May 24th, 2005, http://www.newscientist.com/article.ns?id=dn7418

Entanglement entropy, black holes and holography

(hep-th/0510021)

with Stephen D. H. Hsu

Why to study the black hole entropy?

aspect	gives insights into
BH statistical mechanics	quantum gravity
Hawking radiation	unitarity
…	…

Candidates for the origin of the BH entropy:

model-dependent:	model-independent:
superstring theory	topology of instantons
M-theory	bifurcating Killing horizons
loop quantum gravity	statistical entropy of a thermal gas
…	…
	entaglement

Black holes

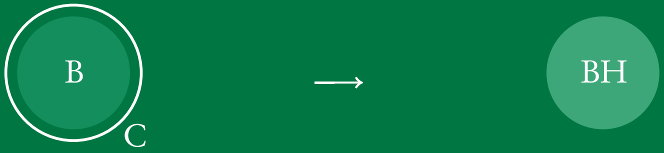
Bekenstein and Hawking:

- mass: $M \sim R$
- temperature: $T \sim R^{-1}$
- entropy: $S_{\mathrm{BH}} \sim R^2 \sim A$

Usually entropy is extensive: $S \sim V$.

Why are black holes so different? .... A mystery.

A mystery with a radical consequence



- matter in a region B is almost ready to collapse to a BH
- add a thin shell C
- B + C collapses to a BH
- entropy does not decrease: $S_B + S_C \leq S_{BH} \sim A$
- $S_C \geq 0 \quad \Rightarrow \quad S_B \lesssim A$
- $\mathcal{N} \sim e^S \lesssim e^A$ microscopic degrees of freedom
- usually $\mathcal{N} \sim e^V$

Does gravity reduce dimensionality from d to $d-1$? AdS/CFT

’t Hooft’s argument for holography

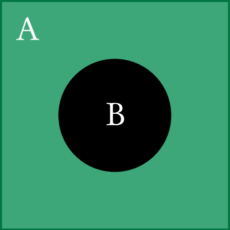
How many QFT states in a region of size R and energy E?

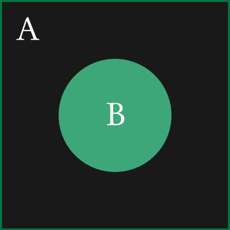
- the most probable state: gas at some temperature T
- energy: $E \sim R^3 T^4$
- entropy: $S_{\text{th}} \sim R^3 T^3$
- no gravitational collapse: $E \lesssim R$
- temperature: $T \lesssim R^{-1/2}$
- entropy: $S_{\text{th}} \lesssim R^{3/2} \sim A^{3/4}$
- a single black hole ($S_{\text{th}} \sim A$) is the limit
- degrees of freedom: $\dim \mathscr{H} = \mathscr{N} \lesssim e^A$

Radical proposal: to describe physics in V it is enough to have one degree of freedom (one bit of information) per Planck area of ∂V.

Entanglement entropy

pure ρ_{AB}





$$\rho_A = \mathrm{tr}_B\, \rho_{AB}$$
$$S_A = -\mathrm{tr}_A\, \rho_A \ln \rho_A$$

$$\rho_B = \mathrm{tr}_A\, \rho_{AB}$$
$$S_B = -\mathrm{tr}_B\, \rho_B \ln \rho_B$$

$$S_A = S_B$$

(not immediately obvious, but can be proved)

Maximally entangled state

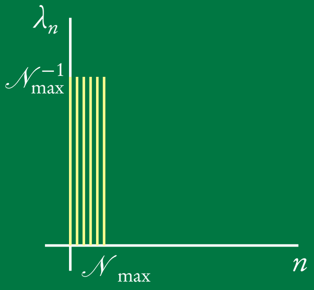
Erroneous argument :

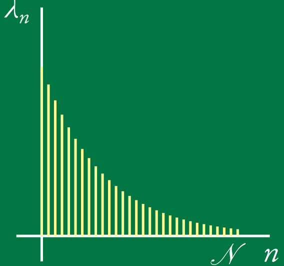
$S_{\mathrm{A}} = S_{\mathrm{B}}$, so entropy depends only on common AB boundary, yielding $S = f(\mathrm{Area}) \sim \mathrm{Area}$.

Counterexample:

$$\max_{\{\lambda_n\}} S \quad \text{with} \quad \sum_n \lambda_n = 1 \quad \text{gives} \quad \lambda_n = \mathscr{N}^{-1}, \quad \max S = \ln \mathscr{N} \sim V$$

Collapse and the entanglement entropy





- begin with states of maximal entropy and impose no-gravitational-collapse condition
- equal probabilities for $n \leq \mathcal{N}_{\max} \ll \mathcal{N}$ states
- $S \sim A^{3/4}$

- maximize the entropy subject to the collapse condition
- the more energetic state, the lower its probability
- $S \lesssim A^{3/4}$

Maximizing the entropy

- maximize: $S = -\sum_n \lambda_n \ln \lambda_n$
- constraints: $\sum_n \lambda_n = 1$ and $\sum_n \lambda_n \epsilon_n = A$
- use the method of Lagrange multipliers:
  $\tilde{S} = -\sum_n \lambda_n \ln \lambda_n + \alpha(\sum_n \lambda_n - 1) - \beta(\sum_n \lambda_n \epsilon_n - A)$
- maximization gives $\lambda_n = Z^{-1} e^{-\beta\epsilon_n}$
  extremal system is thermal
  β is determined by the average total energy condition $\langle E \rangle = R$
- to evaluate partition function $Z(\beta) = \sum_n e^{-\beta\epsilon_n}$,
  take continuous limit $Z(\beta) = \int d\epsilon\, \nu(\epsilon) e^{-\beta\epsilon}$
  and use the saddle point method: $\ln Z(\beta) \approx \ln \nu(\epsilon_*) - \beta\epsilon_*$
  the saddle point: $\epsilon = \epsilon_*(\beta)$, a solution of $d \ln \nu(\epsilon)/d\epsilon = \beta$
- find: $\max S \approx \ln \nu(\epsilon)|_{\epsilon=A}$
  use 't Hooft's result $\epsilon = ER \sim S_{\mathrm{th}}^{4/3} \sim \nu^{4/3}$
  to get $\max S \approx A^{3/4}$
- in d spacetime dimensions: $\max S \approx A^{(d-1)/d}$

Holography interpretations

Old:

States with $\langle H \rangle \gtrsim R$ simply do not exist in the Hilbert space.

New:

The gravitational collapse condition on ψ_{AB} places an upper bound on the entanglement entropy that can be realized from a region B without forming a black hole larger than B itself.

Highly energetic states remain in the theory, but cannot be used to increase the entropy beyond the area of B in Planck units.

Entropy bounds reflect the limitations that gravity imposes on the construction of pure states or density matrices, but do not require a truncation of the Hilbert space itself.