On the origin of probability in quantum mechanics

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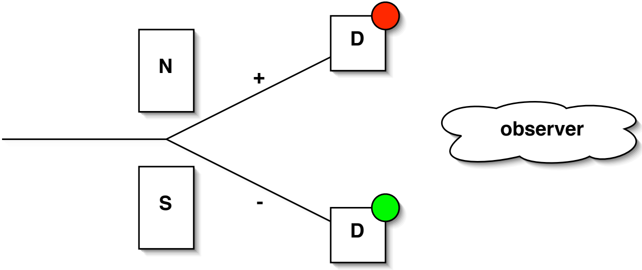
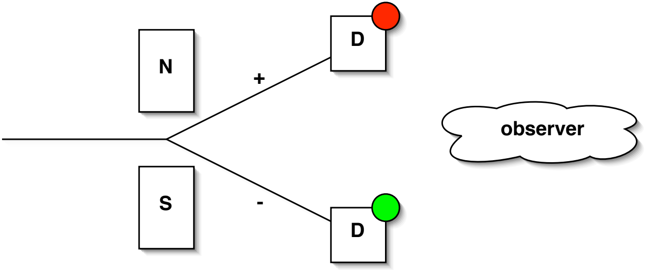
Thought experiment



Stern-Gerlach device (magnet, detector, red/green lights) plus observer.

$$|+\rangle \longrightarrow |up\rangle \otimes |red\rangle \otimes |observed\ red\rangle$$
$$|-\rangle \longrightarrow |down\rangle \otimes |green\rangle \otimes |observed\ green\rangle$$

Thought experiment



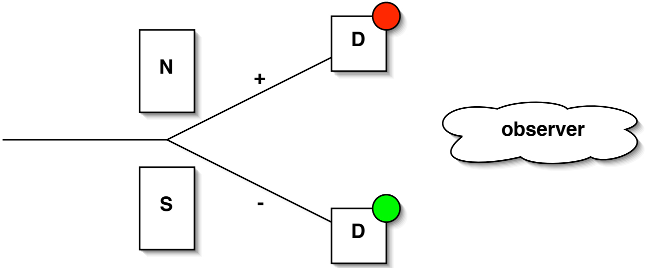
Suppose initial state is a superposition: $|\psi\rangle = c_+|+\rangle + c_-|-\rangle$.

Schrodinger evolution (linear) leads to:

$c_+|up\rangle \otimes |red\rangle \otimes |observed\ red\rangle \ + \ c_-|down\rangle \otimes |green\rangle \otimes |observed\ green\rangle$.

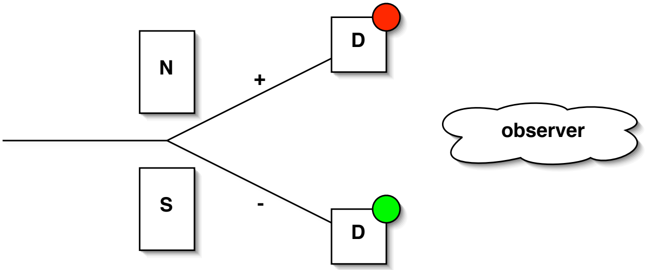
Or does the wavefunction "collapse"?

Thought experiment

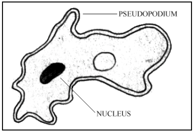


Does it matter if *observer* = *nothing* ? (Or a few atoms? Many atoms?)

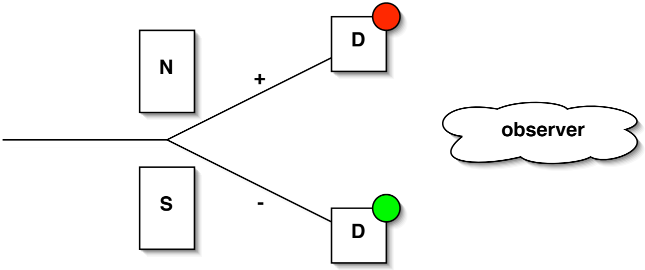
Thought experiment



Does it matter if *observer* = *amoeba* ?

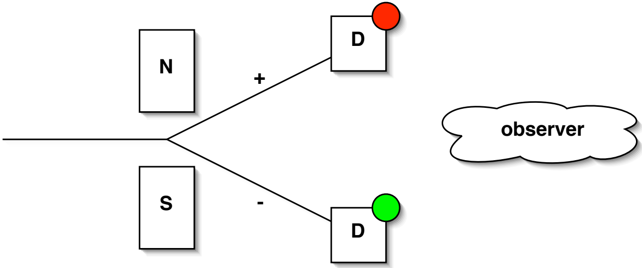


Thought experiment



Does it matter if *observer* = *cat* ?
(Schrodinger's cat?)

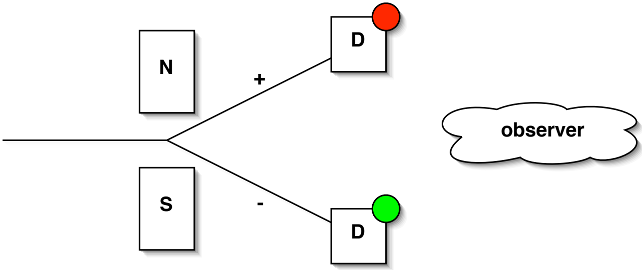
Thought experiment



Does it matter if *observer* = *Gork the robot* ?

(S. Coleman, *Quantum Mechanics, In Your Face!*)

Thought experiment



Does it matter if *observer* = *a scientist* ?

Time evolution in QM

Quantum mechanics, as conventionally formulated, has two types of time evolution:

U: Unitary, deterministic: $\psi(t) = e^{-iHt}\psi(0)$

C: Copenhagen, Collapse: discontinuous, probabilistic

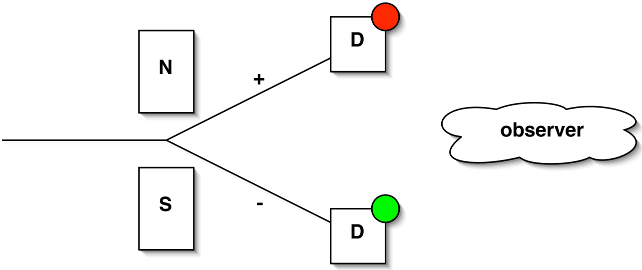
(von Neumann projection: $|\psi\rangle \longrightarrow |a\rangle$, for outcome a.)

Is **C** really necessary? Perhaps **C** is an *apparent* phenomena which emerges from unitary evolution **U**!

H. Everett, 1958

Quantum cosmology. Quantum computers.

Many Worlds Interpretation



= "No collapse" or "Decoherent histories".

Macroscopic beings will *perceive* a collapse due to decoherence of distinct branches.

DeWitt, Hartle, Gell-Mann, Feynman, Hawking, Coleman, Weinberg, Deutsch...

What kind of quantum mechanic are you?

The unexamined life: Gee, never really thought about it! Copenhagen is OK with me.

OK, I thought about it and something IS funny...

Instrumentalist: Copenhagen OK, we're not permitted to interpret physical models beyond experimental predictions. (*How do you feel about quarks?*) Also, we're not permitted to think about the QM evolution of the universe as a whole.

No Collapse: Copenhagen is just obfuscation. It's not a well-defined theory, and Many Worlds is.

QM is incomplete: Many Worlds is too weird for me. QM is incomplete and we'll discover something beyond it someday. (*Well, what are you waiting for? Why waste time on any other problems in physics?*)

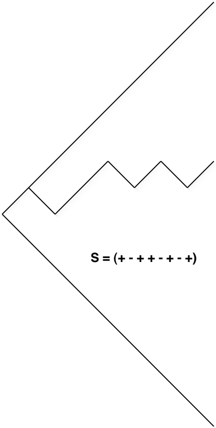
Steve Weinberg, **Einstein's Mistakes**, Physics Today 2005

...Bohr's version of quantum mechanics was deeply flawed, but not for the reason Einstein thought. The Copenhagen interpretation describes what happens when an observer makes a measurement, but the observer and the act of measurement are themselves treated classically. This is surely wrong: Physicists and their apparatus must be governed by the same quantum mechanical rules that govern everything else in the universe. But these rules are expressed in terms of a wavefunction (or, more precisely, a state vector) that evolves in a perfectly deterministic way. So where do the probabilistic rules of the Copenhagen interpretation come from?

Steve Weinberg, **Einstein's Mistakes**, Physics Today 2005

...Considerable progress has been made in recent years toward the resolution of the problem, which I cannot go into here. It is enough to say that neither Bohr nor Einstein had focused on the real problem with quantum mechanics. The Copenhagen rules clearly work, so they have to be accepted. But this leaves the task of explaining them by applying the deterministic equation for the evolution of the wavefunction, the Schrodinger equation, to observers and their apparatus. The difficulty is not that quantum mechanics is probabilistic—that is something we apparently just have to live with. The real difficulty is that it is also deterministic, or more precisely, that it combines a probabilistic interpretation with deterministic dynamics.

Many Worlds Interpretation



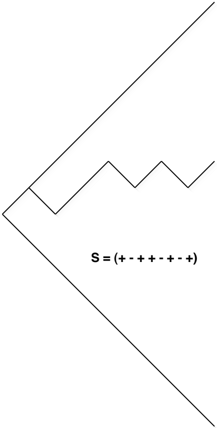
Ensemble state: $\Psi = \otimes_{i=1}^{N} \psi_i = \psi_1 \otimes \psi_2 \cdots \otimes \psi_N$.

Each of 2^N branches or histories S appears in Ψ:

$\Psi = \sum_{\{s_1, \ldots s_N\}} c_{s_1 \ldots s_N} |s_1, \ldots s_N\rangle$

$S = (s_1, s_2, \ldots s_N)$, $s_i = \pm$.

Born rule



How does Born rule ($p_\pm = |c_\pm|^2$) arise in No Collapse interpretation?

Each observer perceives a particular history S, but all branches are possible. Structure of tree completely independent of $c_\pm$!

Multiplicity of branches S dominated by $n_+ \approx n_- = N/2$.

On most branches S, physicists would not have deduced the Born rule!

Born rule

$S = (+ + - + + - + - - + \cdots)$

Let $n = n_+$, $f = f_+ = n/N$ and $p = p_+ = |c_+|^2$.

Assume Born rule. Use it to compute the norm squared of branches based on statistical properties of S, such as number of + outcomes, n.

$$\sum_{n + \text{outcomes}} |c_{s_1,\dots,s_N}|^2 \; = \; P(n) \; = \; \binom{N}{n} p^n (1-p)^{N-n}$$

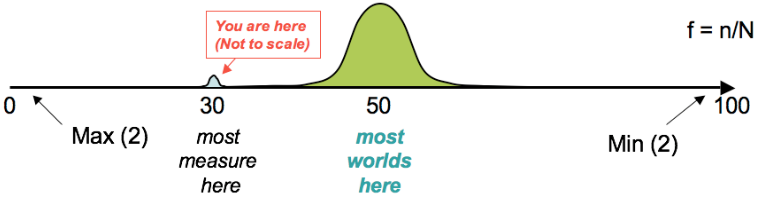
1) combinatorial factor $\binom{N}{n}$ peaked at $n = N/2$ or $f = 1/2$.

2) individual probability factor $p^n(1-p)^{N-n}$ peaked at $n = 0, N$ or $f = 0, 1$.

Competition between (1) and (2): $P(n)$ peaked at $n = pN$ or $f = p$.

Born rule

Example: $p = p_+ = .3$



Key observation: Born rule probability is just the norm squared for a set of outcomes. In conventional QM, probability = measure. Branches which are rare according to Born rule must have small norm.

Norm of Ψ dominated by $|s_1, \ldots, s_N\rangle$ with $f \approx p$ (fraction of + values = .3).

Branches with $f \neq p$ are called maverick worlds (Everett).

Born rule

For $N \to \infty$, states with $f \neq p$ (maverick worlds) have *measure zero*. Only states with fraction of + values given by Born rule have non-zero norm.

Since zero norm states are unphysical, we have derived the Born rule within the No Collapse interpretation.

(Everett, DeWitt, Hartle)

Our horizon is finite

Can $N \to \infty$?

Our causal horizon is finite. Decoherence times are fast, but not zero. Hence N is finite.

For N finite, does the argument still work?

Maverick worlds have small, but non-zero, norm. We have no justification for removing components of Ψ with small but non-zero norm. In fact, in the absence of wavefunction collapse, the norm of a subcomponent $|s_1, \ldots, s_N\rangle$ plays no role in quantum mechanics!

Frequentist vs Bayesian notion of probability.

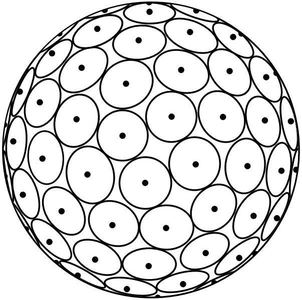
Discrete quantum state space

Quantum mechanics + gravity = minimum length.

No meaning to distance less than the Planck length: l_P
(Calmet, Graesser and Hsu 2004.)

This suggests a discreteness to quantum state space, or Hilbert space, in abuse of terminology. (Buniy, Hsu and Zee 2005.)

Discrete quantum state space



Consider rotation of Stern-Gerlach device by small angle ϵ. If ϵ sufficiently small, no component is displaced by more than a Planck length l_P. The corresponding spin eigenstates are indistinguishable.

We are motivated to consider a *minimum norm* in Hilbert space.

$$|\psi - \psi'| < \epsilon \quad \rightarrow \quad |\Psi - \Psi'| < \sqrt{N}\epsilon.$$

Born rule from discrete state space

Suppose we are allowed to drop components of Ψ below some threshold in norm $\sqrt{N}\epsilon \ll 1$.

The collective norm squared of all maverick states $|\delta, N\rangle$ with frequency deviation $|\delta| = |f - p|$ greater than δ_0 is

$$\sum_{|\delta|>\delta_0} \langle \delta, N|\delta, N\rangle \approx 2N \int_{p+\delta_0}^{\infty} df \; P(fN)\,. \tag{1}$$

where $P(fN) \approx [2\pi N p(1-p)]^{-1/2} \exp\left[-\frac{N(f-p)^2}{2p(1-p)}\right]$.

Requiring that this collective norm squared is less than $N\epsilon^2$ yields

$$\delta_0 > N^{-1/2}\left[2p(1-p)|\ln(N\epsilon^2)|\right]^{1/2}. \tag{2}$$

Born rule from discrete state space

If, for finite N, an experimenter could measure all N outcomes which define his branch of the wavefunction, he might find a deviation from the predicted Born frequency $f = p$ as large as

$$|\ln(N\epsilon^2)|^{1/2}$$

standard deviations (i.e., measuring the deviation in units of $N^{-1/2}$).

Born rule from discrete state space

An experimenter is unlikely to be able to measure more than a small fraction of the outcomes that determine his branch.

A particular branch of the wavefunction is specified by the sequence of outcomes $S = (s_1, s_2, \ldots, s_N)$.

N is the *total number* of decoherent outcomes on a branch, so it is typically enormous – at least Avogadro's number if the system contains macroscopic objects such as an experimenter.

Born rule from discrete state space

The experimental outcomes available to test Born's rule will be some much smaller number $N_* \ll N$ corresponding to some subset of the s_i directly related to the experiment.

Any deviation from the Born rule of order $N^{-1/2}$ will be well within the experimental statistical error of order $N_*^{-1/2}$.

The Born rule will be observed to hold in all the branches which remain after truncation due to discreteness.

Some numbers

Let $\epsilon \sim 10^{-100}$ – a tiny discreteness scale!

Assume $N \sim 10^{160} \sim H^{-4}$ in Fermis.

Then $N\epsilon^2 \sim 10^{-40}$ and

$$|\ln(N\epsilon^2)|^{1/2} \sim 10$$

so the predicted deviation from Born rule is 10 standard deviations *for the entire universe*.

Unless experimenters can measure more than $N/(10)^2$ of all N branchings, their statistical accuracy will not be enough to exclude this deviation.

Summary

Original derivation of Born rule in No Collapse interpretations is flawed: only applies when $N = \infty$ (strict frequentist limit), *but our universe is finite*.

Very small discreteness of quantum state space is enough to restore the result – if $\exists$ minimum norm in Hilbert space, detectable Maverick worlds can be excluded.